1.9 Distance & Midpoint Formulas; Circles

The Distance Formula

The distance, \( d \), between the points \((x_1, y_1)\) and \((x_2, y_2)\) in the rectangular coordinate system is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]

To compute the distance between two points, find the square of the difference between the \(x\)-coordinates plus the square of the difference between the \(y\)-coordinates. The principal square root of this sum is the distance.

**EXAMPLE 1** Using the Distance Formula

Find the distance between \((-1, 4)\) and \((3, -2)\). Express the answer in simplified radical form and then round to two decimal places.
**Check Point 1** Find the distance between \((-1, -3)\) and \((2, 3)\). Express the answer in simplified radical form and then round to two decimal places.

**The Midpoint Formula**
Consider a line segment whose endpoints are \((x_1, y_1)\) and \((x_2, y_2)\). The coordinates of the segment’s midpoint are

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

To find the midpoint, take the average of the two \(x\)-coordinates and the average of the two \(y\)-coordinates.

**EXAMPLE 2** Using the Midpoint Formula
Find the midpoint of the line segment with endpoints \((1, -6)\) and \((-8, -4)\).
Check Point 2 Find the midpoint of the line segment with endpoints (1, 2) and (7, −3).

Definition of a Circle
A circle is the set of all points in a plane that are equidistant from a fixed point, called the center. The fixed distance from the circle’s center to any point on the circle is called the radius.

The Standard Form of the Equation of a Circle
The standard form of the equation of a circle with center \((h, k)\) and radius \(r\) is

\[
(x - h)^2 + (y - k)^2 = r^2.
\]
EXAMPLE 3 Finding the Standard Form of a Circle’s Equation
Write the standard form of the equation of the circle with center \((0, 0)\) and radius 2.
Graph the circle.

Check Point 3 Write the standard form of the equation of the circle with center \((0, 0)\) and radius 4.
Check Point 4 Write the standard form of the equation of the circle with center (0, -6) and radius 10.

EXAMPLE 5 Using the Standard Form of a Circle’s Equation to Graph the Circle

a. Find the center and radius of the circle whose equation is

\[(x - 2)^2 + (y + 4)^2 = 9.\]

b. Graph the equation.

c. Use the graph to identify the relation’s domain and range.
Check Point 5

a. Find the center and radius of the circle whose equation is
\[(x + 3)^2 + (y - 1)^2 = 4.\]

b. Graph the equation.

c. Use the graph to identify the relation’s domain and range.

The General Form of the Equation of a Circle

The **general form of the equation of a circle** is
\[x^2 + y^2 + Dx + Ey + F = 0,\]
where \(D, E,\) and \(F\) are real numbers.

We can convert the general form of the equation of a circle to the standard form
\[(x - h)^2 + (y - k)^2 = r^2.\] We do so by completing the square on \(x\) and \(y.\) Let’s see how this is done.
EXAMPLE 6  Converting the General Form of a Circle’s Equation to Standard Form and Graphing the Circle

Write in standard form and graph: $x^2 + y^2 + 4x - 6y - 23 = 0$.

Assignment

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