Rational Function
Any function that appears in fraction form

\[ \frac{f(x)}{g(x)} \]

ex: \[ \frac{x+1}{x+2} \] or \[ \frac{x^2-9}{x+4} \]

Asymptotes
General Rule: Asymptotes are dotted lines that graphs of equations approach but don't usually cross.

- Horizontal
- Vertical
- Slanted
Vertical Asymptotes

Vertical dotted lines

Special: Graphs \textbf{NEVER} cross or touch vertical asymptotes.

Why? There's only one way to get a V.A.: Divide by zero.

\[ y = \frac{1}{x-1} \]

\( \chi = 1 \) is a V.A.

\[
\begin{array}{c}
\chi = 1 \Rightarrow 0 \\
\chi = 1 \\
\end{array}
\]

ex: Find all vertical asymptotes for:

\[
y = \frac{3}{x^2 + x - 6} = 0
\]

\((x+3)(x-2) = 0\)

\(x = -3, 2\)
ex: Find all vertical asymptotes for:
\[ y = \frac{x+3}{x^2-9} \]
\[ y = \frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3} = 0 \]
\[ x+3 = 0 \]
\[ x = -3 \rightarrow \text{hole} \]
\[ x = 3 \rightarrow \text{v. a.} \]

Discontinuities
- Removable \(\rightarrow\) hole
- Non Removable \(\rightarrow\) vertical asymptotes

ex: Find all vertical asymptotes for:
\[ y = \frac{x-1}{x^2-1} \]
\[ y = \frac{x-1}{(x+1)(x-1)} = \frac{1}{x+1} = 0 \]
\[ x = -1 \rightarrow \text{v. a.} \]
\[ x = 1 \rightarrow \text{hole} \]

\[ f(x) = \frac{(x+1)(x-1)}{(x+1)(x-1)(x+3)(x+5)(x-5)} \]

holes
- \(x = -3\)
- \(x = 3\)

\[ x = -1 \rightarrow \text{v. a.} \]
Horizontal Asymptotes

Horizontal dotted lines

Graphs *can* cross over it, but *eventually* will settle down to it.

ex.

\[ f(x) = \frac{3x^2 + 4}{2x + 1} \]

To find any horizontal asymptotes, compare the degree of the numerator to the degree of the denominator.

\[ H.A. \rightarrow y = 0 \quad (x\text{-axis}) \]

\[ T < B \rightarrow H.A. \quad y = 0 \quad (x\text{-axis}) \]
\[ T = B \rightarrow H.A. \quad y = \frac{L.C.}{L.C.} \]
\[ T > B \rightarrow \text{No H.A.} \]

\( \Rightarrow \) Possible S.A.

\[ \Rightarrow T = B + 1 \]
### Slant Asymptotes

Dotted lines that go at an angle

Graphs can cross over if they want...

To find a slant asymptote:

Check to see if the top is exactly 1 degree higher than the bottom

\[
\frac{x^2 - 4}{x+1} \quad \text{will have a S.A.}
\]

\[
\frac{x^3 - 4}{x+1} \quad \text{No S.A.}
\]

To find the slant asymptote, use synthetic division to divide the fraction and toss the remainder

\[x^2 - 4 \div (x+1) = x - 1, \text{ remainder } 0\]

\[y = x - 1\]

S.A.
Find the slant asymptote for:
\[ y = \frac{x^2 - 4x - 5}{x - 3} \]

\[
\begin{array}{c|ccc}
3 & 1 & -4 & -5 \\
& 3 & -3 & \\
\hline
1 & -1 & \frac{-2}{3} \\
\end{array}
\]

\[ S.A. \rightarrow y = x - 1 \]

\[
\frac{f(x)}{g(x)} = \frac{2x^2 + 3x - 7}{2x + 1}
\]

\[
\begin{array}{c|ccc}
-\frac{1}{2} & 2 & 3 & -7 \\
& 1 & -1 & \\
\hline
& 2 & 2 & \Delta \checkmark
\end{array}
\]

\[ y = 2x + 2 \]

Assignment

page 377

1, 3, 7, 25, 29, 31, 37, 39, 41, 85
Find the Horizontal Asymptotes for the following functions

1) \( \frac{7x^2 - 4}{2x^2 - 9x^3} \)

2) \( \frac{4 + x}{7x^3 - 1} \)

3) \( \frac{2x + 1}{\sqrt{3x^2 - 1}} \)