4-2 Unit Circle

Definitions of the Trigonometric Functions in Terms of a Unit Circle

If $t$ is a real number and $P = (x, y)$ is a point on the unit circle that corresponds to $t$, then

\[
\begin{align*}
\sin t &= y \\
\csc t &= \frac{1}{y}, y \neq 0 \\
\cos t &= x \\
\sec t &= \frac{1}{x}, x \neq 0 \\
\tan t &= \frac{y}{x}, x \neq 0 \\
\cot t &= \frac{x}{y}, y \neq 0.
\end{align*}
\]
Angles on the Unit Circle

A circle with a radius of 1.

The Unit Circle Hand Trick
4.2 Unit Circle

January 18, 2017

EXAMPLE 3  Finding Values of the Trigonometric Functions

at $t = \frac{\pi}{4}$

Find $\sin \frac{\pi}{4}$, $\cos \frac{\pi}{4}$, and $\tan \frac{\pi}{4}$.
4.2 Unit Circle

Check Point 3  Find \( \csc \frac{\pi}{4}, \sec \frac{\pi}{4}, \) and \( \cot \frac{\pi}{4}. \)

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Even and Odd Trigonometric Functions

The cosine and secant functions are even.

\[
\cos(-t) = \cos t \quad \sec(-t) = \sec t
\]

The sine, cosecant, tangent, and cotangent functions are odd.

\[
\sin(-t) = -\sin t \quad \csc(-t) = -\csc t \\
\tan(-t) = -\tan t \quad \cot(-t) = -\cot t
\]

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EXAMPLE 4  Using Even and Odd Functions to Find Values of Trigonometric Functions

Find the value of each trigonometric function:

a. \( \cos\left(-\frac{\pi}{4}\right) \)  

b. \( \tan\left(-\frac{\pi}{4}\right) \)
Check Point 4  Find the value of each trigonometric function:

a. \( \sec \left( -\frac{\pi}{4} \right) \)    
b. \( \sin \left( -\frac{\pi}{4} \right) \).

### Fundamental Identities

**Reciprocal Identities**

\[
\begin{align*}
\sin t &= \frac{1}{\csc t} \\
\cos t &= \frac{1}{\sec t} \\
\tan t &= \frac{1}{\cot t} \\
\csc t &= \frac{1}{\sin t} \\
\sec t &= \frac{1}{\cos t} \\
\cot t &= \frac{1}{\tan t}
\end{align*}
\]

**Quotient Identities**

\[
\begin{align*}
\tan t &= \frac{\sin t}{\cos t} \\
\cot t &= \frac{\cos t}{\sin t}
\end{align*}
\]
4.2 Unit Circle

Example 5  Using Quotient and Reciprocal Identities

Given \( \sin t = \frac{2}{5} \) and \( \cos t = \frac{\sqrt{21}}{5} \), find the value of each of the four remaining trigonometric functions.

Check Point 5  Given \( \sin t = \frac{2}{3} \) and \( \cos t = \frac{\sqrt{5}}{3} \), find the value of each of the four remaining trigonometric functions.
Pythagorean Identities

\[ \sin^2 t + \cos^2 t = 1 \quad 1 + \tan^2 t = \sec^2 t \quad 1 + \cot^2 t = \csc^2 t \]

**EXAMPLE 6** Using a Pythagorean Identity

Given that \( \sin t = \frac{3}{5} \) and \( 0 < t < \frac{\pi}{2} \), find the value of \( \cos t \) using a trigonometric identity.

**Check Point 6** Given that \( \sin t = \frac{1}{2} \) and \( 0 < t < \frac{\pi}{2} \), find the value of \( \cos t \) using a trigonometric identity.
Periodic Properties of the Sine and Cosine Functions

\[ \sin(t + 2\pi) = \sin t \quad \text{and} \quad \cos(t + 2\pi) = \cos t \]

The sine and cosine functions are periodic functions and have period \(2\pi\).

Periodic Properties of the Tangent and Cotangent Functions

\[ \tan(t + \pi) = \tan t \quad \text{and} \quad \cot(t + \pi) = \cot t \]

The tangent and cotangent functions are periodic functions and have period \(\pi\).

**EXAMPLE 7** Using Periodic Properties

Find the value of each trigonometric function:

a. \(\sin \frac{9\pi}{4}\)

b. \(\tan \left(-\frac{5\pi}{4}\right)\).
Check Point 7 Find the value of each trigonometric function:

a. \( \cot \frac{5\pi}{4} \)

b. \( \cos \left( -\frac{9\pi}{4} \right) \)

Assignment

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