4.3 Right Triangle Trig

The word

SOHCAHTOA (pronounced: so-cah-toe-ah)

may be helpful in remembering the definitions for sine, cosine, and tangent.

\[
\begin{align*}
\text{Sine} & : \frac{\text{Opposite}}{\text{Hypotenuse}} \\
\text{Cosine} & : \frac{\text{Adjacent}}{\text{Hypotenuse}} \\
\text{Tangent} & : \frac{\text{Opposite}}{\text{Adjacent}}
\end{align*}
\]

**EXAMPLE 1** Evaluating Trigonometric Functions

Find the value of each of the six trigonometric functions of \( \theta \) in Figure 4.32.

![Figure 4.32](image-url)
Check Point 1  Find the value of each of the six trigonometric functions of $\theta$ in the figure.

\[ \triangle ABC \]

\[ a = 3 \]
\[ b = 4 \]

\[ \theta \]

Example 2  Evaluating Trigonometric Functions

Find the value of each of the six trigonometric functions of $\theta$ in Figure 4.33.

\[ \triangle ABC \]

\[ c = 3 \]
\[ a = 1 \]
\[ b \]

Figure 4.33
Check Point 2 Find the value of each of the six trigonometric functions of $\theta$ in the figure. Express each value in simplified form.

Function Values for Some Special Angles

EXAMPLE 3 Evaluating Trigonometric Functions of $45^\circ$

Use Figure 4.34 to find $\sin 45^\circ$, $\cos 45^\circ$, and $\tan 45^\circ$.

FIGURE 4.34 An isosceles right triangle
Check Point 3 Use Figure 4.34 to find \( \csc 45^\circ, \sec 45^\circ, \) and \( \cot 45^\circ. \)

\[
\begin{array}{c}
\sqrt{2} \\
1 \\
45^\circ \\
1
\end{array}
\]

**FIGURE 4.34** An isosceles right triangle

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**EXAMPLE 4** Evaluating Trigonometric Functions of \(30^\circ\) and \(60^\circ\)

Use Figure 4.35 to find \( \sin 60^\circ, \cos 60^\circ, \sin 30^\circ, \) and \( \cos 30^\circ. \)

\[
\begin{array}{c}
2 \\
30^\circ \\
\sqrt{3} \\
60^\circ \\
1
\end{array}
\]

**FIGURE 4.35** \(30^\circ-60^\circ-90^\circ\) triangle
Check Point 4 Use Figure 4.35 to find $\tan 60^\circ$ and $\tan 30^\circ$. If a radical appears in a denominator, rationalize the denominator.

Any pair of trigonometric functions $f$ and $g$ for which

$$f(\theta) = g(90^\circ - \theta) \quad \text{and} \quad g(\theta) = f(90^\circ - \theta)$$

are called cofunctions. Using Figure 4.36, we can show that the tangent and cotangent are also cofunctions of each other. So are the secant and cosecant.

Cofunction Identities

The value of a trigonometric function of $\theta$ is equal to the cofunction of the complement of $\theta$. Cofunctions of complementary angles are equal.

$$\sin \theta = \cos(90^\circ - \theta) \quad \cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta) \quad \cot \theta = \tan(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta) \quad \csc \theta = \sec(90^\circ - \theta)$$

If $\theta$ is in radians, replace $90^\circ$ with $\frac{\pi}{2}$. 
EXAMPLE 5 Using Cofunction Identities

Find a cofunction with the same value as the given expression:

a. \( \sin 72^\circ \)  
   b. \( \csc \frac{\pi}{3} \)


Check Point 5  Find a cofunction with the same value as the given expression:

a. \( \sin 46^\circ \)  
   b. \( \cot \frac{\pi}{12} \)
EXAMPLE 6  Problem Solving Using an Angle of Elevation

Sighting the top of a building, a surveyor measured the angle of elevation to be $22^\circ$. The transit is 5 feet above the ground and 300 feet from the building. Find the building's height.
Check Point 6 The irregular blue shape in Figure 4.39 represents a lake. The distance across the lake, \(a\), is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?

![Figure 4.39](image)

If two sides of a right triangle are known, an appropriate trigonometric function can be used to find an acute angle \(\theta\) in the triangle. You will also need to use an inverse trigonometric key on a calculator. These keys use a function value to display the acute angle \(\theta\). For example, suppose that \(\sin \theta = 0.866\). We can find \(\theta\) in the degree mode by using the secondary inverse sine key, usually labelled \(\text{SIN}^{-1}\). The key \(\text{SIN}^{-1}\) is not a button you will actually press. It is the secondary function for the button labeled \(\text{SIN}\).

**Many Scientific Calculators:**

\[
\text{SIN}^{-1} \quad \text{SIN}
\]

Pressing \(2\text{nd SIN}\) accesses the inverse sine key, \(\text{SIN}^{-1}\).

**Many Graphing Calculators:**

\[
\text{SIN}^{-1} \quad \text{SIN} \quad .866 \quad \text{ENTER}
\]

The display should show approximately 59.997, which can be rounded to 60. Thus, if \(\sin \theta = 0.866\), then \(\theta \approx 60^\circ\).
EXAMPLE 7  Determining the Angle of Elevation

A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to the nearest degree.

Check Point 7  A flagpole that is 14 meters tall casts a shadow 10 meters long. Find the angle of elevation of the sun to the nearest degree.
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1, 5, 9, 11, 13, 21, 25, 29, 31, 33, 35, 39, 53, 55, 57, 59